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Electromagnetic nondiffracting pulses in lossless isotropic plasmalike media

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We introduce a scheme for describing electromagnetic nondiffracting pulses propagating in isotropic and lossless media characterized by a plasma-like refractive index. A family of nondiffracting waves in a dispersive medium is analytically derived in the form of a generalization of X waves propagating in vacuum. It is also shown how the ratio between pulse width and plasma length has a crucial effect on the pulse dynamics.

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The first theoretical attempt to reduce diffractive spreading of monochromatic fields dates back to Stratton [1], while their first experimental observation was reported by Durnin [2]. In the polychromatic domain, limited-diffraction pulses have been predicted by Lu and Greenleaf [3–5] in the form of X wave, a class of nondiffracting solutions of the free-space scalar wave equation. More recently, theoretical investigations have been reported both in acoustics [6–9] and electromagnetism [10–19], together with remarkable effort aimed at generating and detecting nondiffracting pulses [20–24]. A number of applications have been proposed like, e.g., medical real-time imaging [25], optical microlithograhy [26], dispersion compensation [27–29], and generation of long plasma fibers for guiding intense pulses [30,31].

As far as dispersive media are concerned, schemes have been considered [32–36] for describing nondiffracting pulses. However, while analytical examples of e.m. nondiffractive pulses in vacuum are available in the form of X waves, no closed-form expressions of nondiffracting pulses in dispersive media are yet been presented in the literature.

In this paper, we investigate nondiffracting pulses propagating in isotropic plasmas. In particular, we develop a formal scheme to describe their behavior, and we find out a family of analytical solutions which can be considered the dispersive generalization of X waves.

The complex analytic signal associated to any cartesian component of an arbitrary electromagnetic pulse propagating (along the positive z axis) in a homogeneous and dispersive medium is given by

$$\hat{f}(\mathbf{r}_\perp,z,t) = \int_0^{+\infty} d\omega \int d^2\mathbf{k}_\perp e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp} e^{i(k_z z-\omega t)} \widetilde{F}(\mathbf{k}_\perp,\omega), \quad (1)$$

where $\mathbf{r}_\perp = x\hat{\mathbf{e}}_x + y\hat{\mathbf{e}}_y$, $\mathbf{k}_\perp = k_x\hat{\mathbf{e}}_x + k_y\hat{\mathbf{e}}_y$, $d^2\mathbf{k}_\perp = dk_x dk_y$, $k_z = \sqrt{\omega^2 n^2(\omega)/c^2 - |\mathbf{k}_\perp|^2}$, $n(\omega)$ is the refractive index, and $\widetilde{F}$ is an arbitrary well-behaved function of its arguments. Nondiffracting pulses are wave packets satisfying the shape-invariance condition $\hat{f}(\mathbf{r}_\perp,z,t) = \hat{f}(\mathbf{r}_\perp,z-Vt)$, which states that the pulse rigidly travels with velocity V along the z axis. In order to specialize Eq. (1) to describe nondiffracting pulses, all the plane-wave components of the packet must satisfy the relation $\omega = k_z V$. This implies that, in the $(\mathbf{k}_\perp,\omega)$ space, a surface Σ (if it exists) is selected to which the integration in Eq. (1) has to be restricted. It is worth noting that the shape of such a surface depends on the speed V and on the refractive index and, hence, on the dispersive properties of the medium. As an example, in vacuum the surface Σ_{vac} is the cone $\omega = \eta V|\mathbf{k}_\perp|$ (see Fig. 1), with $\eta = (V^2/c^2 - 1)^{-1/2}$, and its existence requires that $V > c$ expressing the "superluminaly" of nondiffracting pulses. In an arbitrary dispersive medium it is obvious that, whenever the imaginary part of the refractive index is not negligible, the earlier surface cannot be defined so that nondiffracting pulses do not exist; this is consistent with the fact that, in a highly lossy medium, a pulse cannot propagate undisturbed. Therefore, the integration in Eq. (1) has to be additionally restricted to those parts

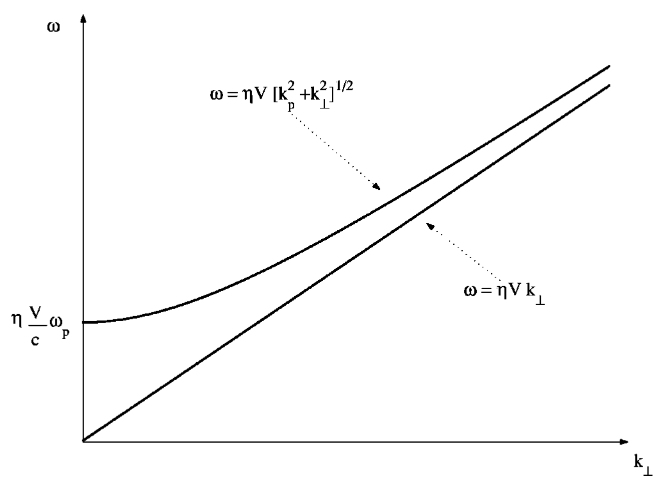

FIG. 1. Relations between ω and $k_\perp = |\mathbf{k}_\perp|$ of a nondiffracting pulse propagating in vacuum $(\omega = \eta V k_\perp)$ and in a plasma $[\omega = \eta V(k_p^2 + k_\perp^2)^{1/2}]$.

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of the surface Σ that are sufficiently far from regions surrounding the resonance frequencies of the medium.

Let us consider a homogeneous and dispersive medium whose refractive index is

$$n(\omega) = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}, \tag{2}$$

where ω is the frequency and ω_p is a characteristic plasma frequency of the medium. The main example of applicability of Eq. (2) is that of a lossless and isotropic plasma composed by electrons and heavy ions, where $\omega_p = \sqrt{4\pi N_0 e^2/m}$, N_0, e, and m being the electron density, charge, and mass. We note that Eq. (2) is also a suitable model for the high frequency tail of the refractive index of most dielectrics and metals, since it holds whenever the field frequencies are much greater than the greatest resonance frequency of the material [37]. Let us now look for nondiffracting pulses in a medium whose refractive index is of the form given by Eq. (2). The surface $\Sigma_{\rm pla}$, resulting from the diffraction-free condition $\omega = k_z V$, is given by $\omega = \eta V(k_p^2 + |\mathbf{k}_\perp|^2)^{1/2}$ (see Fig. 1), where $k_p = \omega_p/c$. As in vacuum, this surface exists only if $V > c$, implying that nondiffracting pulses are superluminal entities also in plasmas. The surface $\Sigma_{\rm pla}$ is a revolution hyperboloid lying above the cone $\Sigma_{\rm vac}$ and asymptotically touching it. The vertex is on the ω axis at $\omega = (\eta V/c)\omega_p$, implying that the whole frequency band of a nondiffracting pulse is above the plasma frequency ω_p, an expected and necessary condition for the reality of $n(\omega)$ in Eq. (2). The existence of this frequency threshold is the major responsible for the differences between nondiffracting pulses in vacuum and plasmas. The restriction of the integration domain in Eq. (1) to the surface $\Sigma_{\rm pla}$ is obtained by requiring that

$$\widetilde{F}(\mathbf{k}_\perp, \omega) = \tilde{f}(\mathbf{k}_\perp)\,\delta(\omega - V\eta\sqrt{k_p^2 + |\mathbf{k}_\perp|^2}), \tag{3}$$

where $\tilde{f}$ is an arbitrary function of $\mathbf{k}_\perp$ and $\delta(\xi)$ is the Dirac delta function. Inserting Eq. (3) into Eq. (1) and performing the integral over ω we readily get

$$\hat{f}(\mathbf{r}_\perp, Z) = \int d^2\mathbf{k}_\perp e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp} e^{i\eta Z\sqrt{k_p^2+|\mathbf{k}_\perp|^2}}\tilde{f}(\mathbf{k}_\perp), \tag{4}$$

where $Z = z - Vt$ is the longitudinal coordinate in a frame rigidly traveling with the pulse. Equation (4) yields the most general expression for a nondiffracting pulse in a plasma. The physical meaning of the spectrum $\tilde{f}(\mathbf{k}_\perp)$ is obtained by evaluating Eq. (4) at $Z=0$ and inverting the resulting Fourier integral, thus obtaining

$$\tilde{f}(\mathbf{k}_\perp) = \frac{1}{(2\pi)^2}\int d^2\mathbf{r}_\perp e^{-i\mathbf{k}_\perp\cdot\mathbf{r}_\perp}\hat{f}(\mathbf{r}_\perp, 0), \tag{5}$$

which is the two dimensional Fourier transform of the field at the waist plane $Z=0$. Note that, setting $k_p=0$ in Eq. (4), we get

$$\hat{f}(\mathbf{r}_\perp, Z) = \int d^2\mathbf{k}_\perp e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp} e^{i\eta Z|\mathbf{k}_\perp|}\tilde{f}(\mathbf{k}_\perp), \tag{6}$$

which describes nondiffracting pulses propagating in vacuum [14], as expected. We note that the existence of the plasma length $L_p = k_p^{-1}$ gives rise to a phenomenology of nondiffracting pulses. In order to understand the effect of the length L_p on the structure of the pulse, let us consider its transverse width w at $Z=0$. Suppose that $w \ll L_p$ so that, as a consequence of Eq. (5), the spectrum $\tilde{f}(\mathbf{k}_\perp)$ is vanishing only outside the circle $|\mathbf{k}_\perp| \gtrsim w^{-1} \gg k_p$. This implies that in the integrand of Eq. (4) k_p can be neglected with respect to $|\mathbf{k}_\perp|$, so that the pulse closely resembles the one in Eq. (6). We conclude that, for narrow pulses, the plasma does not effectively alter the structure of a nondiffracting pulse propagating in vacuum. From a geometrical point of view, this is related to the fact that the spectrum of a narrow pulse "explores" a vast region of the hyperboloid $\Sigma_{\rm pla}$, so that the major spectral contribution is due to regions so far from the origin that $\Sigma_{\rm pla}$ can be approximately replaced by the vacuum cone $\Sigma_{\rm vac}$. In the opposite situation $w \gg L_p$, we conversely have that the spectrum $\tilde{f}(\mathbf{k}_\perp)$ is not negligible only inside the circle $|\mathbf{k}_\perp| \lesssim w^{-1} \ll k_p$ so that a sort of *paraxial* approximation can be done by expanding the square root in Eq. (4) up to the first relevant order, thus getting

$$\hat{f}(\mathbf{r}_\perp, Z) = e^{i\eta k_p Z}\int d^2\mathbf{k}_\perp e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp} e^{i\eta Z/2k_p|\mathbf{k}_\perp|^2}\tilde{f}(\mathbf{k}_\perp). \tag{7}$$

The structure of this nondiffracting pulse is extremely different from that of a standard nondiffracting pulse propagating in vacuum and geometrically this is a consequence of the considerable difference between $\Sigma_{\rm pla}$ and $\Sigma_{\rm vac}$ in the neighborhood of $\mathbf{k}_\perp = 0$, which is the relevant spectral region of a wide pulse. It is interesting to note that the field in Eq. (7) exhibits a structure formally coincident with that of a monochromatic paraxial beam propagating in vacuum (after the substitutions $\omega/c \to k_p/\eta$ and $z \to z - Vt$). As a consequence, the field does not show the usual X-shaped profile typical of a large class of nondiffracting pulses in vacuum. The earlier discussion proves that the plasma length L_p is a key parameter defining two opposite limiting regimes ($w \ll L_p$ and $w \gg L_p$) for nondiffracting pulses in plasmas.

Let us now consider the family of nondiffracting pulses whose spectrum is given by

$$\tilde{f}_n(\mathbf{k}_\perp) = \bar{f}\frac{s^2}{2\pi}(s\sqrt{k_p^2 + |\mathbf{k}_\perp|^2})^{n-1} e^{-s\sqrt{k_p^2+|\mathbf{k}_\perp|^2}}, \tag{8}$$

where n is a positive integer and $\bar{f}$ and s are two real constants. Inserting Eq. (8) into Eq. (4), changing variables according to $k_x = (h^2 - k_p^2)^{1/2}\cos\theta$ and $k_y = (h^2 - k_p^2)^{1/2}\sin\theta$, and performing the integral over θ we get

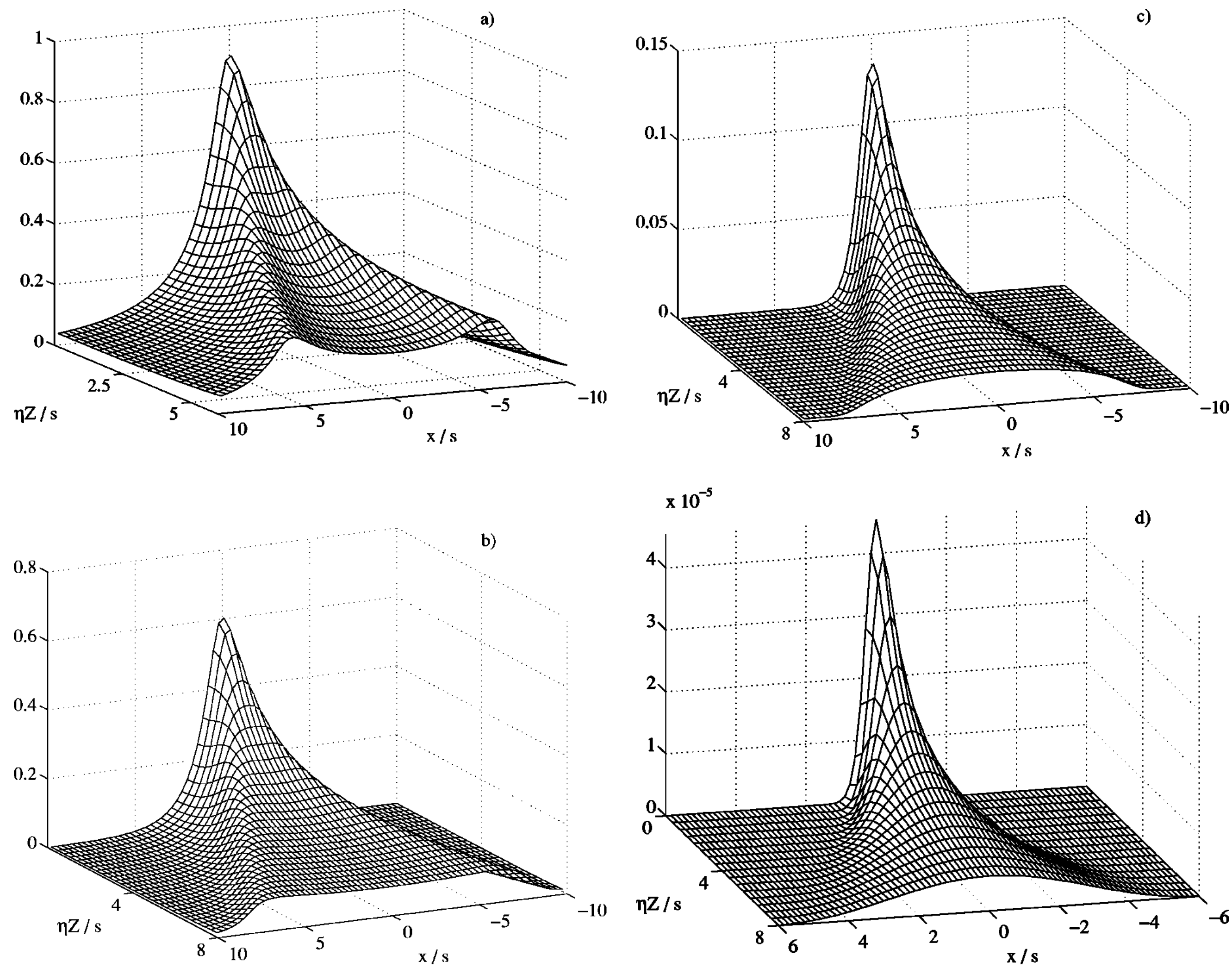

FIG. 2. Plot of the modulus of $f_0/\overline{f}$ from Eq. (11) as a function of x/s and $\eta Z/s$ at $y=0$ for various plasma lengths: (a) $L_p=10s$; (b) $L_p=2s$; (c) $L_p=0.5s$; (d) $L_p=0.1s$. Note the X-shaped profile of plot (a) and the typical paraxial features of plot (d).

$$\hat{f}_n(\mathbf{r}_\perp,Z) = \overline{f}s^{n+1}\int_{k_p}^{\infty} dh e^{-(s-i\eta Z)h} h^n J_0(|\mathbf{r}_\perp|\sqrt{h^2-k_p^2}). \quad (9)$$

Exploiting the properties of the Laplace transform, we readily obtain [38]:

$$\hat{f}_n(\mathbf{r}_\perp,Z) = \overline{f}\frac{\partial^n}{\partial\zeta^n}\left[\frac{e^{-k_p s\sqrt{(1-\zeta)^2+|\mathbf{r}_\perp|^2/s^2}}}{\sqrt{(1-\zeta)^2+\frac{|\mathbf{r}_\perp|^2}{s^2}}}\right]_{\zeta=i\eta Z/s}. \quad (10)$$

This is an exact result and also an *analytical example* of nondiffracting pulses propagating in a dispersive medium. The family of nondiffracting pulses of Eq. (10) can be regarded as the dispersive plasma counterpart of the well-known family of X waves introduced by Lou and Greenleaf [4] since, in the limiting case $k_p\rightarrow 0$, the two families coincide. In order to discuss the main properties of this class of fields, let us consider its first (and also generating) element obtained from Eq. (10) for $n=0$, i.e.:

$$\hat{f}_0(\mathbf{r}_\perp,Z) = \overline{f}\frac{e^{-k_p s\sqrt{(1-i\eta Z/s)^2+|\mathbf{r}_\perp|^2/s^2}}}{\sqrt{\left(1-\frac{i\eta Z}{s}\right)^2+\frac{|\mathbf{r}_\perp|^2}{s^2}}}, \quad (11)$$

so that the parameter s turns out to control the pulse width on its waist at $Z=0$. The influence of the medium is contained in the k_p-dependent exponential, implying $f_0\approx\exp(-k_p|\mathbf{r}_\perp|)$ for $|\mathbf{r}_\perp|\rightarrow+\infty$, so that the pulse transversally falls off faster than the corresponding packet in vacuum. For $s\ll L_p$, i.e., $k_p s \ll 1$, the pulse approaches the vacuum counterpart (the exponential practically being equal to 1). For values of s comparable or larger than L_p the medium influence tends to become relevant, as intuitive, and the pulse loses the typical X-shaped profile. In Fig. 2, we plot the modulus of the normalized field $f_0/\overline{f}$ of Eq. (11) for various plasma lengths. Note that the two arms of the X, typical of narrow pulses, tend to merge for increasing pulse width into a unique structure whose shape closely resembles a paraxial Gaussian beam. It is also interesting to note that a field closely related to Eq. (11) has been considered in a nonlinear regime by Conti [39] in the frame of nondiffracting propagation in Kerr media.

In conclusion we have presented a treatment for describing nondiffracting pulses propagating in lossless and isotropic plasmas. In particular we have obtained a relevant family of exact solutions extending to dispersive media the well-known class of vacuum *X* waves. Our results can be applied to a wide class of dielectrics and metals in the high frequency regime characterized by a refractive index of the form of Eq. (2).

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